Stolarsky-3 Mean Labeling on Triangular Snake Graphs

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Abstract - *Let* *G = (V, E) be a graph with p vertices and q edges . G is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,...,q+1 and each edge e = uv is assigned the distinct labels* ***f****(e = uv) =* $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ *(or)* $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ *then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we investigate the Stolarsky-3 Mean labeling for Double Triangular Snake, Triple Triangular Snake, Four Triangular Snake and Alternative Triple Triangular snake graphs.*

Keywords - *Graph Labeling, Stolarsky-3 Mean Labeling, Triangular Snake Graphs, Double Triangular Snake Graph, Triple Triangular Snake graph , Four Triangular Snake graph.*

I. Introduction

*All graphs in this paper are finite, simple and undirected graphs. We follow Gallian**[1]** for all detailed survey of graph labeling and we refer Harary**[2]** for all other standard terminologies and notations. The concept of "Mean **Labeling"** has been introduced by S.Somasundaram and R.Ponraj in 2004**[3]** and S.Somasundaram, R.Ponrajand S.S. Sandhya introduced the concept of "**Harmonic Mean Labeling of graphs" in[4].** "**Stolarsky-3 mean labeling**" was introduced by S.S. Sandhya, E.Ebin Raja Merly and S.Kavitha in **[5].***

The following definitions are used here for our present study.

Definition 1.1: *A graph G with p vertices and q edges is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2 ,... ,q+1 and each edge e=uv is assigned the distinct labels f(e = uv) =* $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ *(or)* $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ *then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G.*

Definition 1.2: A Triangular Snake T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \le i \le$n-1. That is, every edge of a path is replaced by a triangle C_3.

Definition 1.3: Double Triangular Snake D (T_n) consists of two Triangular snakes that have a common path.

Definition 1.4: Triple Triangular Snake T (T_n) consists of three Triangular snakes that have a common path.

Definition 1.5: An Alternate Triangular Snake A (T_n) is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} alternatively to a new vertexv_i. That is every alternate edge of a path is replaced by C_3.

Definition 1.6: An Alternate Triple Triangular Snake A (T(T_n)) consists of three Alternate Triangular snakes that have a common path.

Definition 1.7: Four Triangular Snake F (T_n) consists of four Triangular snakes that have a common path.

Now we shall use the frequent reference to the following theorems.

Theorem 1.8 [3]: Triangular Snake graph is a Mean graph (S.Somasundaram & R.Ponraj).

Theorem 1.9 [4]: Triangular Snake graph is Harmonic Mean graph (S.Somasundaram, R.Ponraj & S.S. Sandhya).

Theorem 1.10 [6]: Triangular Snake graph is Stolarsky-3 Mean graph (S.S. Sandhya, E.Ebin Raja Merly and S.Kavitha).

II. Main Results

Theorem 2.1: Double Triangular Snake graph $D(T_n)$ is Stolarsky-3 Mean graph.

Proof: Let P_n be the path $u_1, u_2, \ldots, u_n$.

To construct $D(T_n)$, Join u_i and u_{i+1} to two new vertices v_i, w_i, $1 \leq i \leq n-1$.

Define a function **f**: $V(D(T_n)) \rightarrow \{1, 2, \ldots, q+1\}$ by

$f(u_i) = 5i - 4, 1 \leq i \leq n.$

$f(v_i) = 5i - 3, 1 \leq i \leq n-1.$

$f(w_i) = 5i - 2, 1 \leq i \leq n-1.$

Then the edges are labeled with

$f(u_i u_{i+1}) = 5i - 2, 1 \leq i \leq n-1.$

$f(u_i v_i) = 5i - 4, 1 \leq i \leq n-1.$

$f(u_i w_i) = 5i - 3, 1 \leq i \leq n-1.$

$f(v_i u_{i+1}) = 5i - 1, 1 \leq i \leq n-1.$

$f(w_i u_{i+1}) = 5i, 1 \leq i \leq n-1.$

Then the edge labels are distinct.

Hence $D(T_n)$ is Stolarsky-3 Mean graph.

Example 2.2: *The Stolarsky-3 Mean labeling of $D(T_4)$ is given below.*

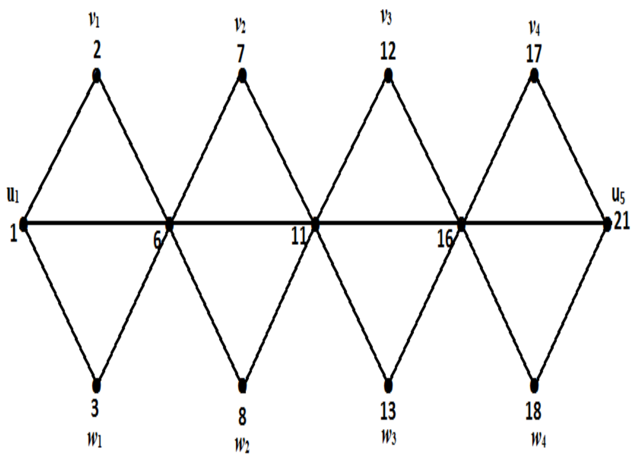

Figure: 1

Theorem 2.3: *Triple Triangular Snake graph $T(T_n)$ is Stolarsky-3 Mean graph.*

Proof: *Let P_n be the path $u_1, u_2, \ldots, u_n$.*

To construct $T(T_n)$, Join u_i and u_{i+1} to three new vertices v_i, w_i and x_i, $1\leq i\leq n-1$.

Define a function $f : V(T(T_n)) \rightarrow \{1,2, \ldots, q+1\}$ by

$f(u_i) = 7i - 6$, $1\leq i \leq n$.

$f(v_i) = 7i - 5$, $1\leq i \leq n-1$.

$f(w_i) = 7i - 4$, $1\leq i \leq n-1$.

$f(x_i) = 7i - 3$, $1\leq i \leq n-1$.

Then the edges are labeled with

$f(u_i u_{i+1}) = 7i - 3$, $1\leq i \leq n-1$.

$f(u_i v_i) = 7i - 6$, $1\leq i \leq n-1$.

$f(u_i w_i) = 7i - 5$, $1\leq i \leq n-1$.

$f(u_i x_i) = 7i - 4$, $1\leq i \leq n-1$.

$f(v_i v_{i+1}) = 7i - 2$, $1\leq i \leq n-1$.

$f(w_i u_{i+1}) = 7i - 1$, $1\leq i \leq n-1$.

$f(x_i u_{i+1}) = 7i$, $1\leq i \leq n-1$.

Then the edge labels are distinct. *Hence $T(T_n)$ is Stolarsky-3 Mean graph.*

Example 2.4: *The Stolarsky-3 Mean labeling of $T(T_4)$ is given below.*

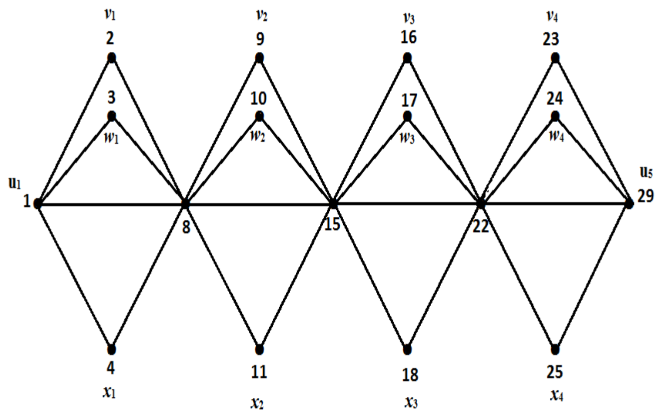

Figure: 2

Theorem 2.5: *Alternate Triple Triangular snake $A(T(T_n))$ is Stolarsky-3 Mean graph.*

Proof: *Let G be the graph $A(T(T_n))$.*

Consider a path $u_1, u_2, \ldots, u_n$.

To construct G, join u_i and u_{i+1} (alternatively) with three new vertices v_i, w_i and x_i, $1\leq i\leq n-1$.

There are two different cases to be considered.

Case(1) *If G is starts from* u_1*, we consider two sub cases.*

Sub case (1) (a) n is odd

Define a function f: V (G)→{ 1,2,…, q+1} by

$$f(u_{2i-1}) = 8i\text{-}7,\ 1\leq i \leq \frac{n-1}{2}+1.$$

$$f(u_{2i}) = 8i,\ 2\leq i \leq \frac{n-1}{2}.$$

$$f(v_i) = 8i\ \text{-}6,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(w_i) = 8i\ \text{-}5,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(x_i) = 8i\ \text{-}4,\ 1\leq i \leq \frac{n-1}{2}.$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = 4i\ ,\ 1\leq i \leq n-1.$$

$$f(u_{2i-1}v_i) = 8\ i\ \text{-}7,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(u_{2i-1}w_i) = 8\ i\ \text{-}6,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(u_{2i-1}x_i) = 8\ i\ \text{-}5,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(v_iu_{2i}) = 8\ i\ \text{-}3,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(w_iu_{2i}) = 8\ i\ \text{-}2,\ 1\leq i \leq \frac{n-1}{2}.$$

$$f(x_iu_{2i}) = 8\ i\ \text{-}1,\ 1\leq i \leq \frac{n-1}{2}.$$

Thus we get distinct edge labels.

The labeling pattern is shown below.

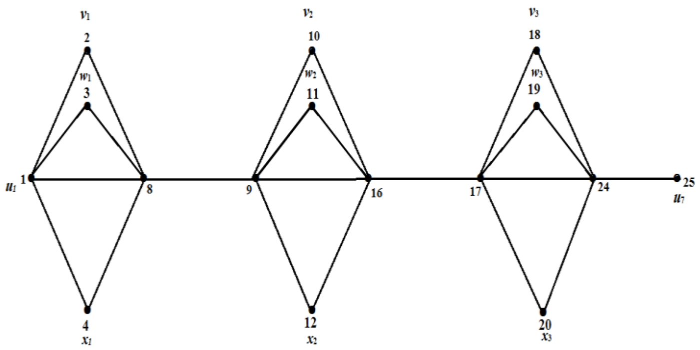

Figure: 3

Sub case (1) (b) n is even

Define a function f: $V(G) \rightarrow \{1,2,\dots,q+1\}$ *by*

$f(u_{2i-1}) = 8i-7, 1 \le i \le \frac{n}{2}$.

$f(u_{2i}) = 8i, \quad 1 \le i \le \frac{n}{2}$.

$f(v_i) = 8i-6, 1 \le i \le \frac{n}{2}$.

$f(w_i) = 8i-5, 1 \le i \le \frac{n}{2}$.

$f(x_i) = 8i-4, 1 \le i \le \frac{n}{2}$.

Then the edges are labeled as

$f(u_i u_{i+1}) = 4i, 1 \le i \le n-1$.

$f(u_{2i-1} v_i) = 8i-7, 1 \le i \le \frac{n}{2}$.

$f(u_{2i-1} w_i) = 8i-6, 1 \le i \le \frac{n}{2}$.

$f(u_{2i-1} x_i) = 8i-5, 1 \le i \le \frac{n}{2}$.

$f(v_i u_{2i}) = 8i-3, 1 \le i \le \frac{n}{2}$.

$f(w_i u_{2i}) = 8i-2, 1 \le i \le \frac{n}{2}$.

$f(x_i u_{2i}) = 8i-1, 1 \le i \le \frac{n}{2}$.

Thus we get distinct edge labels.

The labeling pattern is shown below.

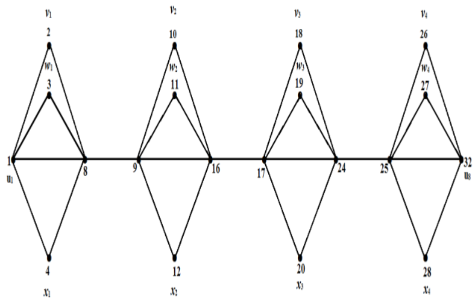

Figure: 4

Case(2) *If G is starts from* u_2 *then we consider two sub cases.*

Sub case (2) (a) n is odd

Define a function f : $V(G) \rightarrow \{1,2,\dots,q+1\}$ *by*

$f(u_{2i-1}) = 8i-7, 1 \le i \le \frac{n-1}{2} + 1$.

$f(u_{2i}) = 8i-6, 1 \le i \le \frac{n-1}{2}$.

$f(v_i)$ = 8i -2, 1≤ $i \leq \frac{n-1}{2}$.

$f(w_i)$ = 8i -1, 1≤ $i \leq \frac{n-1}{2}$.

$f(x_i)$ = 8i -5, 1≤ $i \leq \frac{n-1}{2}$.

Then the edges are labeled as

$f(u_i u_{i+1})$ =4i-3, 1≤ $i \leq n-1$.

$f(u_{2i} v_i)$ =8 i -5, 1≤ $i \leq \frac{n-1}{2}$.

$f(u_{2i} w_i)$ =8 i -4, 1≤ $i \leq \frac{n-1}{2}$.

$f(u_{2i} x_i)$ =8 i -6, 1≤ $i \leq \frac{n-1}{2}$.

$f(v_i u_{2i+1})$ =8 i -1, 1≤ $i \leq \frac{n-1}{2}$.

$f(w_i u_{2i+1})$ =8 i, 1≤ $i \leq \frac{n-1}{2}$.

$f(x_i u_{2i+1})$ =8 i -2, 1≤ $i \leq \frac{n-1}{2}$.

Thus we get distinct edge labels.

The labeling pattern is shown below.

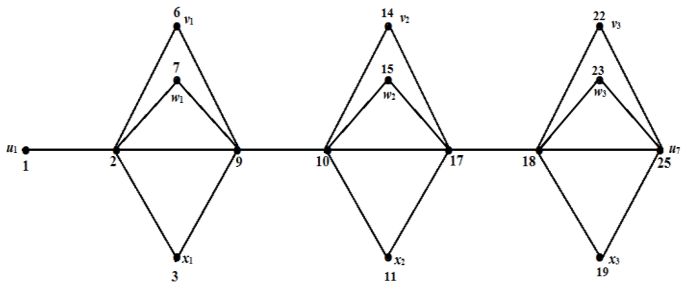

Figure: 5

Sub case (2) (b) n is even

Define a function f: V (G)→{ 1,2,..., q+1} by

$f(u_{2i-1})$ = 8i-7, 1≤ $i \leq \frac{n}{2}$.

$f(u_{2i})$ = 8i-6, 1≤ $i \leq \frac{n}{2}$.

$f(v_i)$ = 8i -2, 1≤ $i \leq \frac{n}{2}$.

$f(w_i)$ = 8i -1, 1≤ $i \leq \frac{n}{2}$.

$f(x_i)$ = 8i -5, 1≤ $i \leq \frac{n}{2}$.

Then the edges are labeled as

$f(u_i u_{i+1})$ =4i-3, 1≤ $i \leq n-1$.

$f(u_{2i}v_i)$ =8 i -5, 1≤ $i \le \frac{n-2}{2}$.

$f(u_{2i}w_i)$ =8 i -4, 1≤ $i \le \frac{n-2}{2}$.

$f(u_{2i}x_i)$ =8 i -6, 1≤ $i \le \frac{n-2}{2}$.

$f(v_iu_{2i+1})$ =8 i -1, 1≤ $i \le \frac{n-2}{2}$.

$f(w_iu_{2i+1})$ =8 i, 1≤ $i \le \frac{n-2}{2}$.

$f(x_iu_{2i+1})$ =8 i -2, 1≤ $i \le \frac{n-2}{2}$.

Thus we get distinct edge labels.

The labeling pattern is shown below.

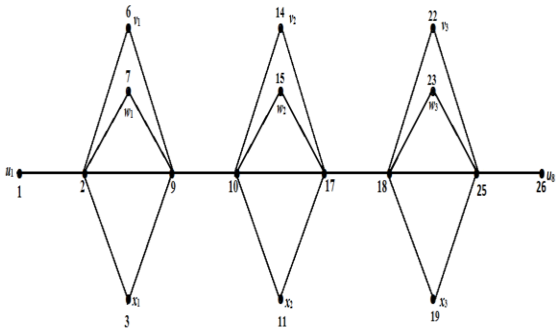

Figure 6

From all the above cases, we can conclude that Alternate Triple Triangular snake A(T(T_n)) is Stolarsky-3 mean graph.

Theorem 2.6: *Four Triangular Snake graph F (T_n) is Stolarsky-3 Mean graph.*

Proof: *Let P_n be the path $u_1, u_2, \ldots, u_n$.*

To construct F(T_n) , join u_i and u_{i+1} with four new vertices v_i ,w_i, v_i' and w_i' ,1≤ i≤ n-1.

*Define a function **f** : V(F(T_n)) →{ 1,2,..., q+1} by*

$f(u_1)$ = 1.

$f(v_1)$ = 6.

$f(w_1)$ = 2.

$f(v_1')$ = 8.

$f(w_1')$ = 4.

$f(u_i)$ =9(i-1), 2≤ $i \le n$.

$f(v_i)$ = 9i -4 , 2≤ $i \le n-1$.

$f(w_i)$ = 9i -8 , 2≤ $i \le n-1$.

$f(v_i')$ = 9i -2 , 2≤ $i \leq n-1$.

$f(w_i')$ = 9i -6, 2≤ $i \leq n-1$.

Then the edges are labeled with

$f(u_i u_{i+1})$ =9i -4, 1≤ $i \leq n-1$.

$f(u_i v_i)$ =9 i -6, 1≤ $i \leq n-1$.

$f(u_i w_i)$ =9 i -8, 1≤ $i \leq n-1$.

$f(u_i v_i')$ =9i -5, 1≤ $i \leq n-1$.

$f(u_i w_i')$ =9i -7, 1≤ $i \leq n-1$.

$f(v_i u_{i+1})$ =9i -1, 1≤ $i \leq n-1$.

$f(w_i u_{i+1})$ =9i -3, 1≤ $i \leq n-1$.

$f(v_i' u_{i+1})$ =9i, 1≤ $i \leq n-1$.

$f(w_i' u_{i+1})$ =9i -2, 1≤ $i \leq n-1$.

Then the edge labels are distinct.

Hence F (T_n) is Stolarsky-3 Mean graph.

Example 2.7: *The Stolarsky-3 Mean labeling of F (T_4) is given below.*

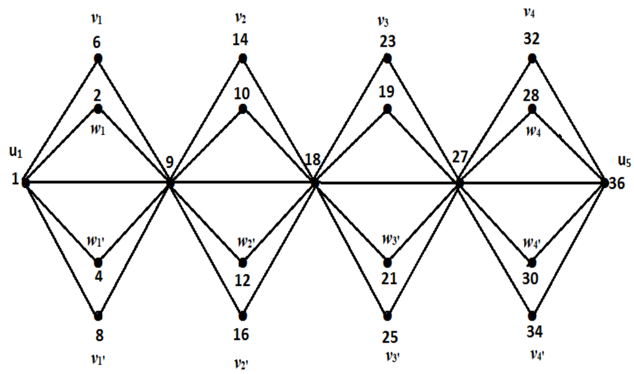

Figure: 7

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